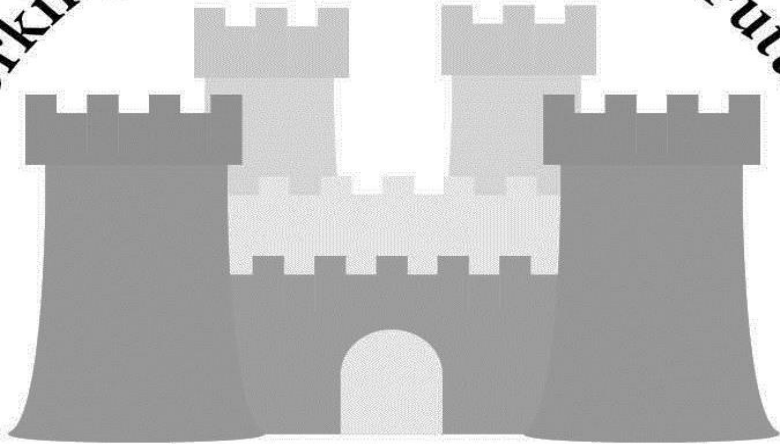


Working Together Building Futures



Burnham Infant School



**Burnham-on-Sea Infant School and
St Andrew's Junior School, Burnham-on-Sea
Maths Calculations Policy
Updated November 2019**



The following calculation policy has been designed to teach children to develop conceptual understanding through the progression of concrete, pictorial and abstract methods and has been agreed by all staff at Burnham-on-Sea Infant School and St. Andrew's Junior School. This policy meets the requirements of the National Curriculum 2014 for the teaching and learning of mathematics. It is also designed to give pupils a consistent and smooth progression of learning in calculations across the schools in line with the mastery approach. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

Using the concrete-pictorial-abstract approach:

Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation:

The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

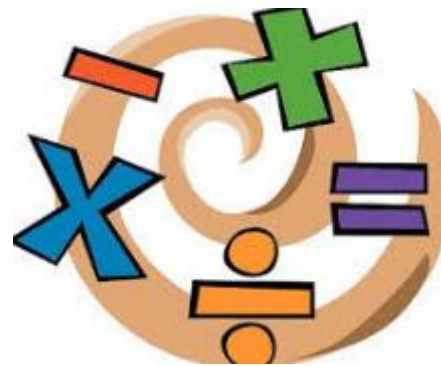
Pictorial representation:

The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation:

The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$.

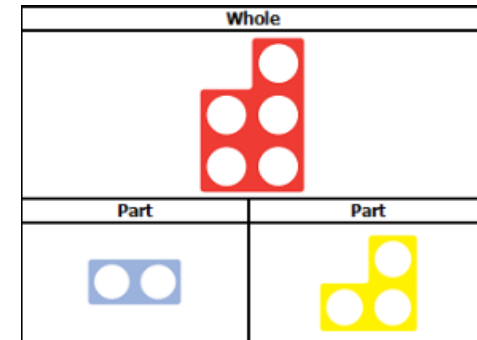
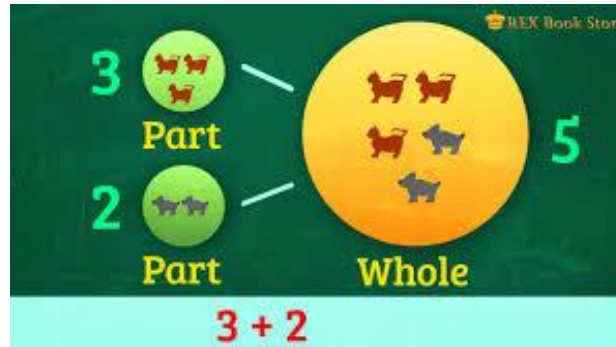
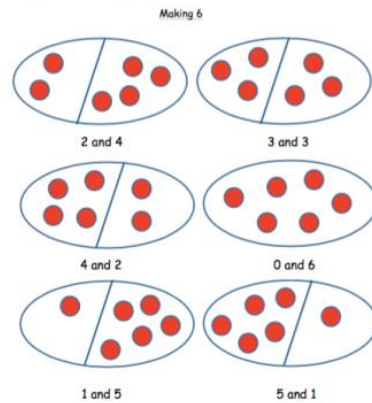
Reception



Addition

Explore part-part whole relationship

They develop ways of recording calculations using pictures

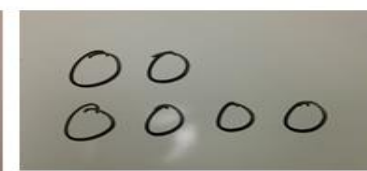


Using the ten frame to support addition of single digits - counting all/combining two groups

	$6 + 4 = 10$
	$4 + 4 = 8$
	$5 + 2 = 7$
	$2 + 4 = 6$

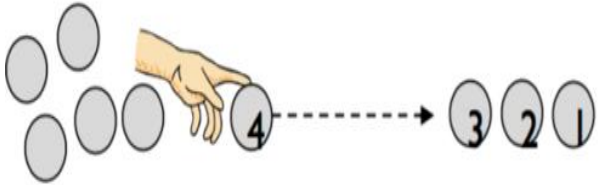
Solving problems using concrete and pictorial images.

Sara has 2 apples.
Jon has 5 apples.
How many apples do they have altogether?
How many more apples does Jon have than Sara?



Subtraction

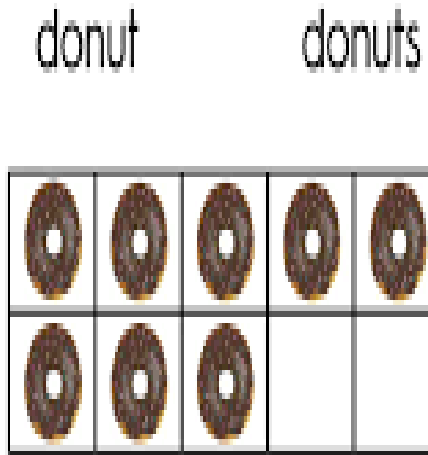
Taking away after counting out practical equipment. .
Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready may record their own calculations

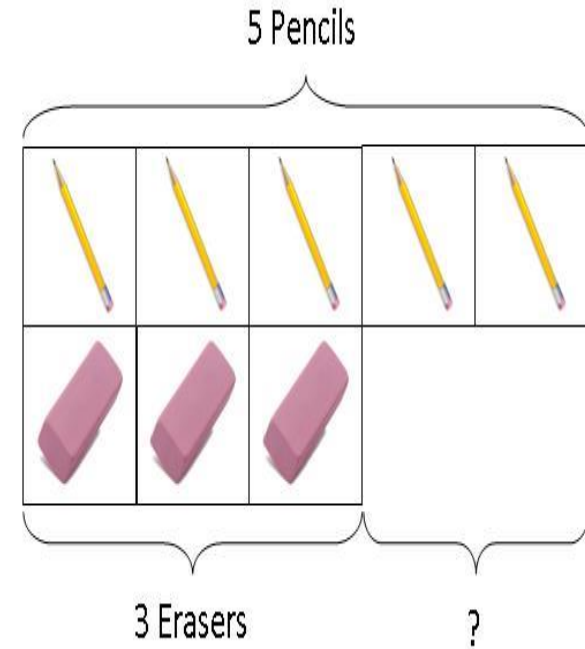
Using the ten frame to support subtraction by taking away.



$$8 - 4 = \underline{\quad}$$

0	1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	---	----

Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?
Solving problems using concrete and pictorial images.



Multiplication

Children will experience equal groups of objects.

They will work on practical problem solving activities involving



There are 6 pairs of socks.
How many socks are there altogether?

Double 2



boots

$$2 + 2 = 4$$

Division

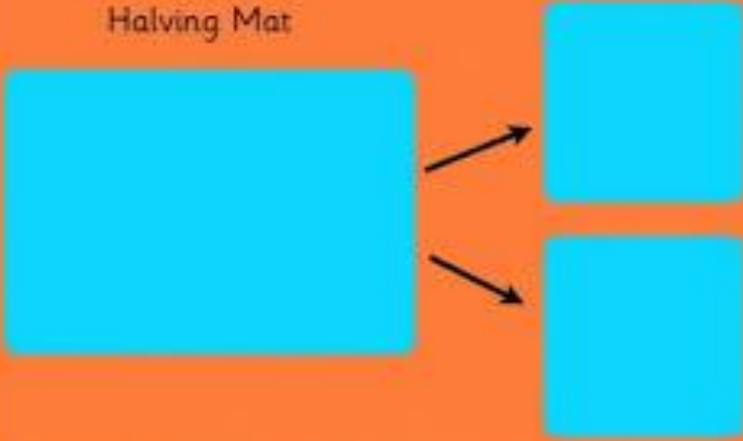


Half is...

10

5

Halving Mat



Year 1



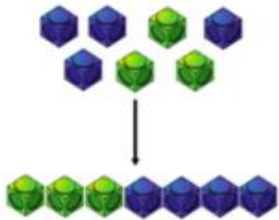
Year 1 - Addition

Key Vocabulary

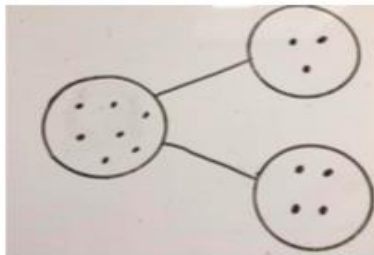
Part, whole, addition, add, addend, sum, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

Combining two parts to make a whole

Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).

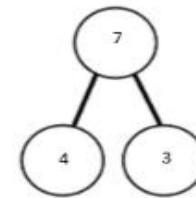


Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.



$$4 + 3 = 7$$

Four is a part, 3 is a part and the whole is seven.



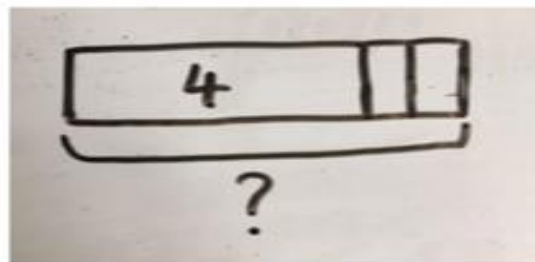
At its base level, addition is **part-part-whole**. The concept that when you **add** two different **values, amounts, objects** etc, the **total amount** you have is your **whole**. This can be expressed like this: **part + part = whole**. It can also be **whole = part + part**.

Counting on to find the total

Counting on using number lines using cubes or Numicon.



A bar model which encourages the children to count on, rather than count all.

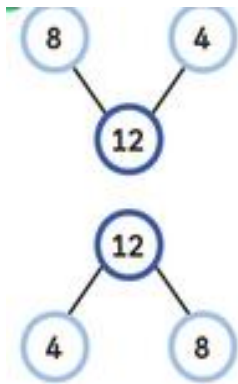


The abstract number line:
What is 2 more than 4?
What is the sum of 2 and 4?
What is the total of 4 and 2?
 $4 + 2$



Children can also find the **total** of two, separate **values** by **counting on** using a =number line. This allows children to see the process of starting with **zero**, recognise this as an amount, and then build towards finding the **whole**.

Related facts (Inverse Operation) including number bonds to 20



$$8 + 4 = 12$$

$$4 + 8 = 12$$

This is a family of addition and subtraction facts.

$$12 - 8 = 4$$

$$12 - 4 = 8$$



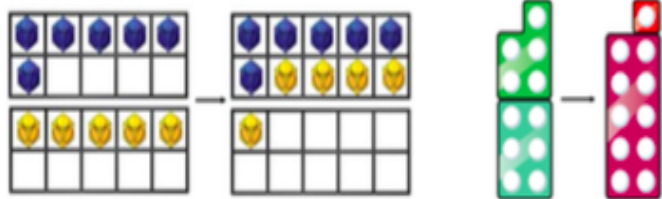
<p>Tens Frame</p> $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$	<p>Part Whole Model</p> $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$	<table border="1"> <tr> <td colspan="2">10</td> </tr> <tr> <td>6</td> <td>4</td> </tr> </table> <p>Bar Model</p> $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$	10		6	4
10						
6	4					

It is important that **addition** and **subtraction** are taught alongside each other as pupils need to see the **relationship** between the facts. **Addition** is the **inverse** (opposite method) of **subtraction**. Using the **part-whole model**, children can see that **part + part = whole** but also **whole - part = part**.

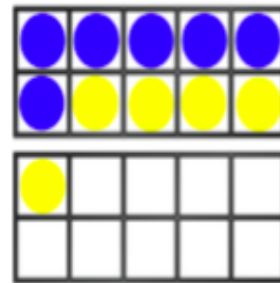
Regrouping to make 10

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

$$6 + 5 = \square + 4$$

Using ten frames and related equipment, this will allow children to use their **number bonds** knowledge to **partition** numbers so they are easier to **add**. This is a mental calculation method that can be applied to larger numbers later on in the Key Stage.

Year 1 - Subtraction

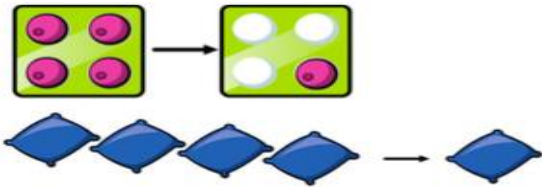
Key Vocabulary

Part, whole, subtraction, subtract, subtrahend, minuend, difference, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit.

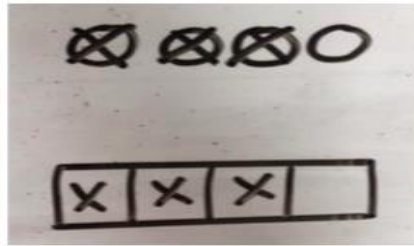
Physically taking away and removing objects from a whole

Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

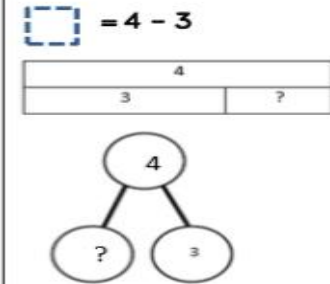
$$4 - 3 = 1$$



Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.



$$4 - 3 =$$



Children need to experience physically taking away and removing **objects** from a **whole**. By doing this, children will come to understand that **subtraction** makes a **number, amount** or **total, smaller** and you have **fewer** items or objects than you did before. This builds the conceptual understanding of what **subtraction** is. It is also important that children understand what you are left with is the **difference**.

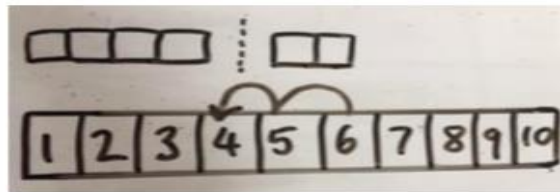
Counting back

Counting back (using number lines or number tracks) children start with 6 and count back 2.

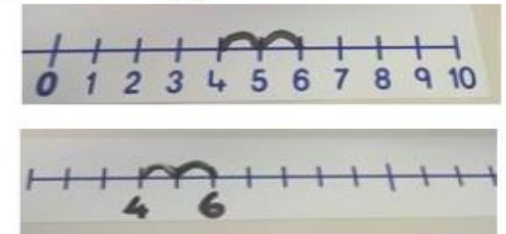
$$6 - 2 = 4$$



Children to represent what they see pictorially e.g.



Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line

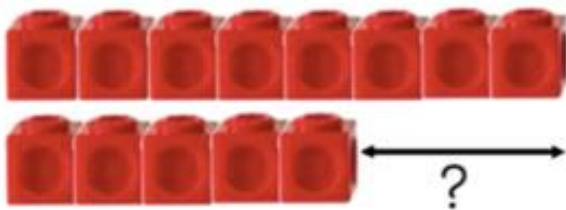


Counting back from the **whole** number to **subtract** can be an invaluable way for children to use their knowledge of the **ordinality** of **number** to find the answer. Numberlines, rulers or fingers can all be used here.

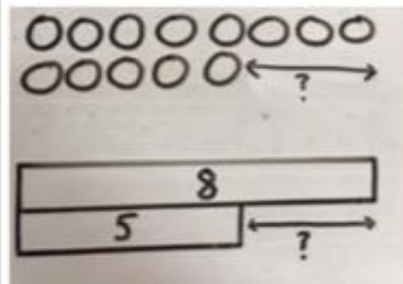
Finding the difference

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



Find the difference between 8 and 5.

$8 - 5$, the difference is

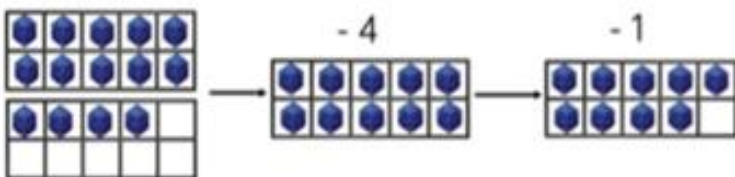
Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Children need to see the **difference** between two **amounts** before they can work this out in the **abstract**. When comparing two sets of objects **the difference** should be clear. Lots of different objects can be used here to enable children to see that **finding the difference** can be done with anything.

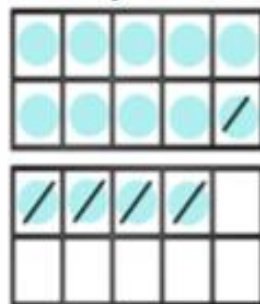
Making 10

Making 10 using ten frames.

$14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

$$14 - 4 = 10$$

$$10 - 1 = 9$$

Much like **counting back to subtract**, **making ten** relies on children's **number bond** knowledge and how to **partition** a number. This method works well when **subtracting** mentally.

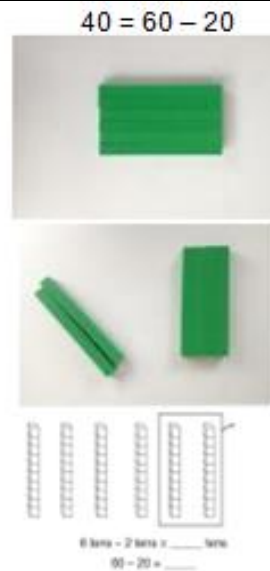
Other ways to subtract



$$20 - 4 = 16$$

When **subtracting** using **Dienes** children should be taught to **regroup** a ten rod for **10 ones** and then **subtract** from those **ones**. Again, this will test **partitioning** knowledge and how to make up a **number** in different ways.

Subtracting multiples of 10



Using the vocabulary of **1 ten**, **2 tens** etc alongside 10, 20, 30 is very important here as pupils need to understand that it is a 10 not a 1 that is being **taken away**.

Year 1 - Multiplication

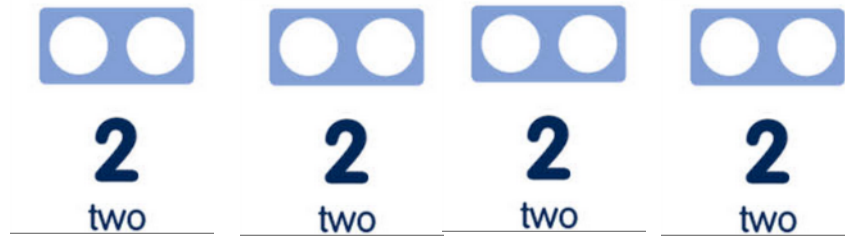
Key Vocabulary

Part, whole, ones, groups, lots of, doubling, multiplicand, multiplier, repeated addition, groups of, lots of, times, columns, rows, longer, bigger, higher etc and times as (big, long, wide ...etc)

Counting in multiples of 2, 5 and 10 from zero

Children should count the number of **groups** on their fingers as they are **skip counting**.

4 groups of 2 = 8

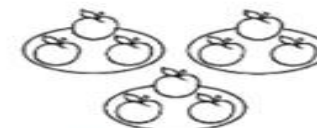


$$2 \times 4 = 8$$

When moving to pictorial/written calculations the vocabulary is important:



This image represents **two groups** of 4 or **4 twice**



How many apples are there altogether?

$$3 + 3 + 3 = 9$$

Solving **multiplication** problems using **repeated addition**

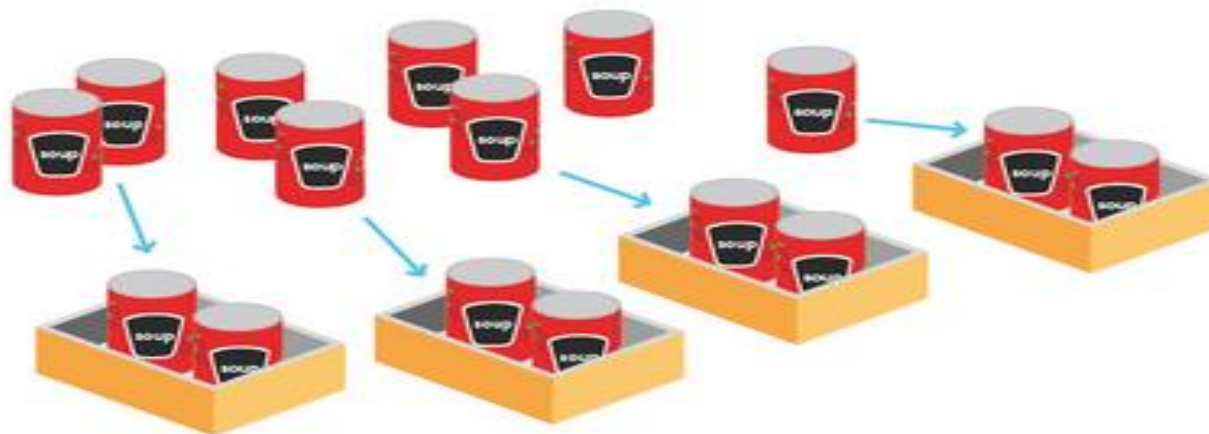
Year 1 - Division

Key Vocabulary

Part, whole, share, share equally, dividing by, dividing into, divisor, dividend, quotient, one each, two each..., group, groups of, lots of, array

Sharing concrete resources practically

1 There are 8 cans.



There are 4 boxes of 2 cans.

Pupils should be taught to **divide** through working practically and **the sharing** should be shown below the whole to familiarize children with the concept of the **whole**.

The language of **whole** and **part part** should be used.

$$8 \div 4 = 2$$

"I have 8 cans. This is my whole number. I want to share them between four boxes. Each box has 2 cans so each box has 2 parts."

Year 2

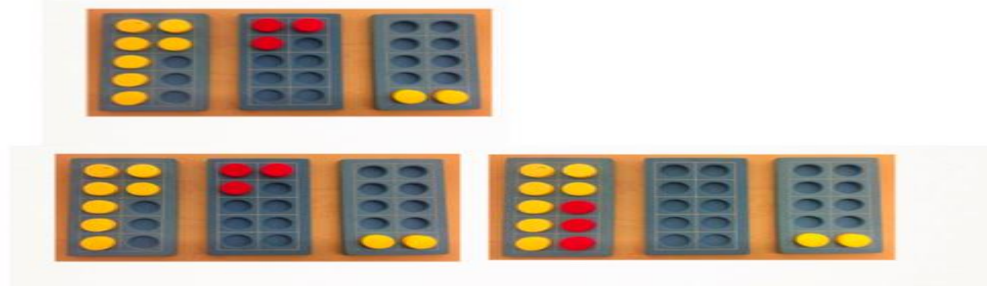


Year 2 - Addition

Key Vocabulary

Part, whole, +, add, addition, more, plus, make, addend, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, tens, ones, partition, Near multiple of 10, tens boundary, more than, one more, two more... ten more...

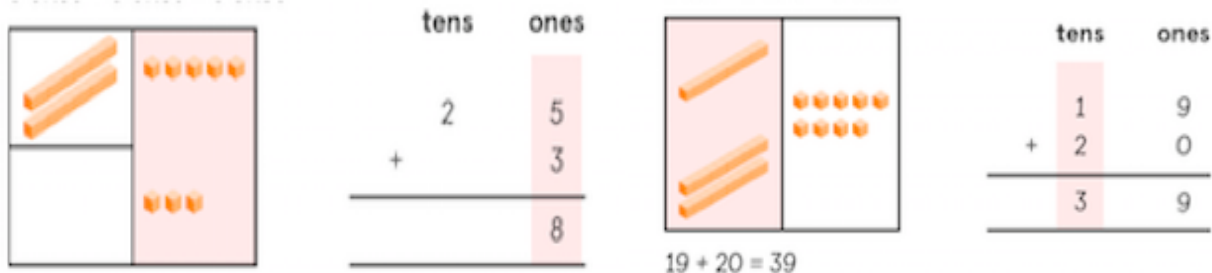
Using concrete objects and pictorial representations to add 3 single digit numbers.



$$7+3+2 = \text{ leads to } 10 + 2 =$$

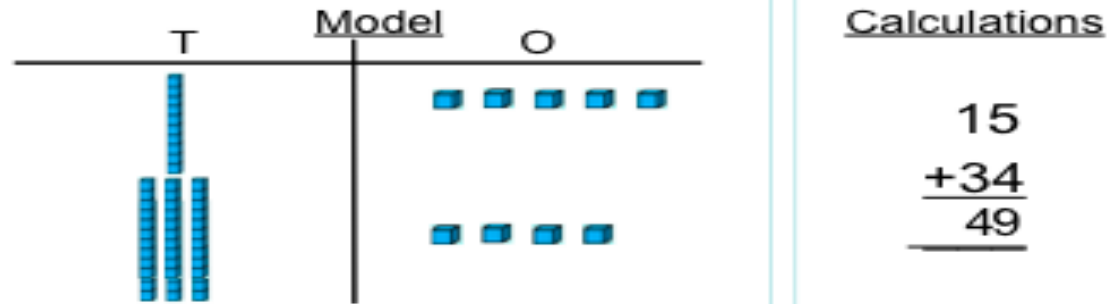
By working with **concrete resources**, children need to be able to see where they can **regroup** to **make 10**. This will link to **number bonds** and how they can always look for **10**.

Using concrete objects and pictorial representations to add a 2 digit number and ones and tens.



Understanding how each number is split into **10s** and **1s** will allow children to understand the method of **addition conceptually** before moving on to the written method.

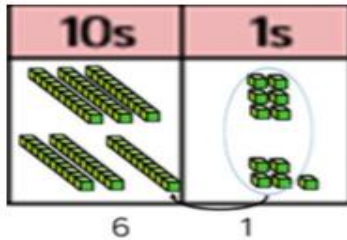
Using concrete objects and pictorial representations to add two 2-digit numbers



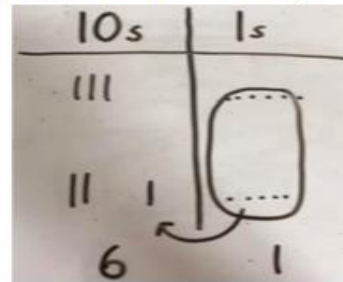
Understanding how each number is split into **10s** and **1s** will allow children to understand the method of **addition conceptually** before moving on to the written method.

Other ways to add

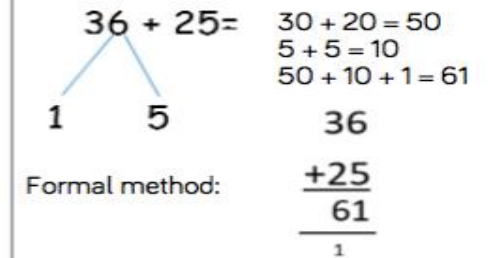
TO + TO using base 10. Continue to develop understanding of partitioning and place value.
 $36 + 25$



Children to represent the base 10 in a place value chart.

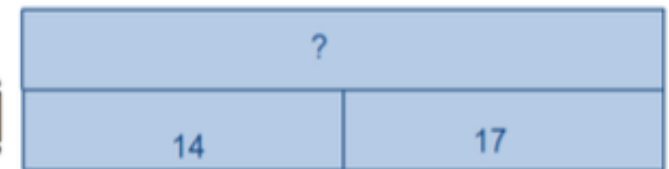
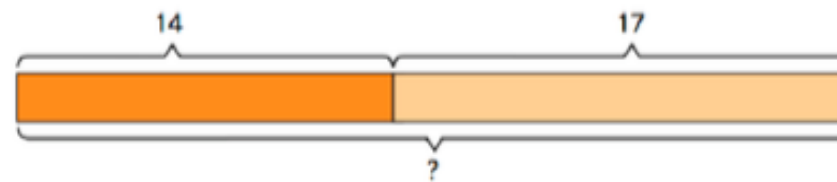


Looking for ways to make 10.



Using the bar to find missing digits

Helen has 14 breadsticks. Her friend has 17. How many do they have altogether?



It is important for children to use the bar in this way to encourage the use of it to aid with **problem solving**. The link between **part part whole** should be made clear. The **length** of each **part** does not always have to be **drawn to scale** either.

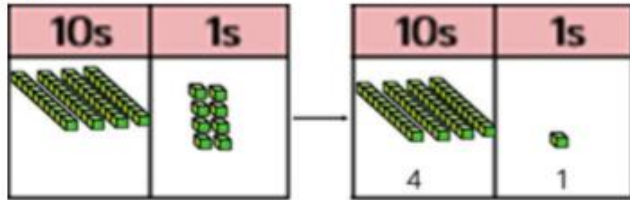
Year 2 - Subtraction

Key Vocabulary

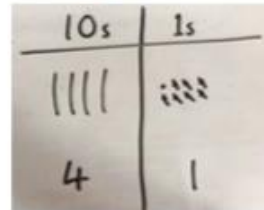
Part, whole, subtraction, subtract, subtrahend, minuend, take away, difference, difference between, minus, tens, ones, partition, Near multiple of 10, tens boundary, Less than, one less, two less... ten less...

Using concrete objects and pictorial representations to subtract a 1-digit number from 2-digit number.

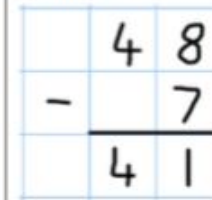
Column method using base 10.
48-7



Children to represent the base 10 pictorially.

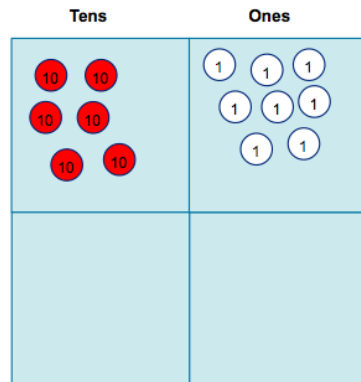


Column method or children could count back 7.

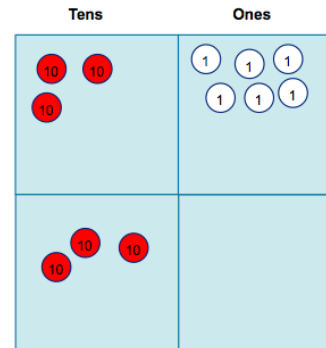


Building on the use of **Dienes** in year 1 to **subtract ones** from a **2-digit number**, children will continue to make the link between **subtraction** making a **whole number smaller**.

Using concrete objects and pictorial representations to subtract a 10s number from 2 digit number.



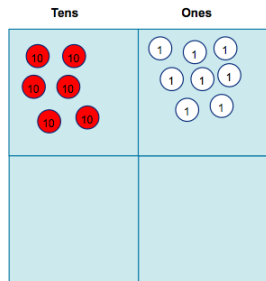
$$\begin{array}{r} 68 \\ - 30 \\ \hline \\ \hline \end{array}$$



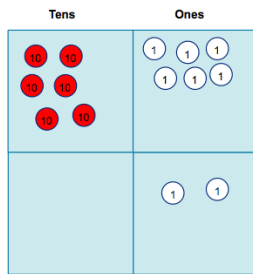
$$\begin{array}{r} 68 \\ - 30 \\ \hline \\ \hline \end{array}$$

Children need the opportunity to see how **taking away a 10s number** from a **2-digit number** does not affect the **ones column**. It is important to discuss why this might be the case. (A **10s number** always ends in 0 ...)

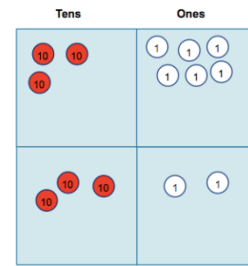
Using concrete objects and pictorial representations to subtract a 2-digit number from 2 digit number.



$$\begin{array}{r} 68 \\ - 32 \\ \hline \hline \end{array}$$



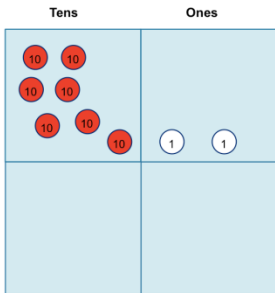
$$\begin{array}{r} 68 \\ - 32 \\ \hline \hline \end{array}$$



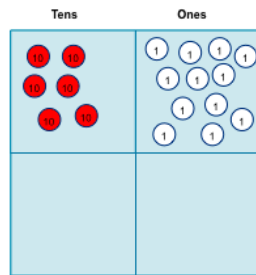
$$\begin{array}{r} 68 \\ - 32 \\ \hline \hline \end{array}$$

Children need to experience **subtracting** a **2-digit number** from a **2-digit number** without having to **exchange** and **regroup**.

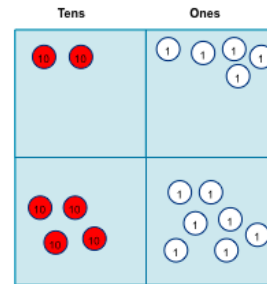
As above but at Greater Depth:



$$\begin{array}{r} 72 \\ - 47 \\ \hline \hline \end{array}$$



$$\begin{array}{r} 72 \\ - 47 \\ \hline \hline \end{array}$$



$$\begin{array}{r} 72 \\ - 47 \\ \hline 25 \end{array}$$

Children working at **greater depth** will need to experience using **concrete resources** and having to **exchange** in order to solve the calculation.

Recognise and use the inverse relationship between addition and subtraction

76	
23	?

?	
23	53

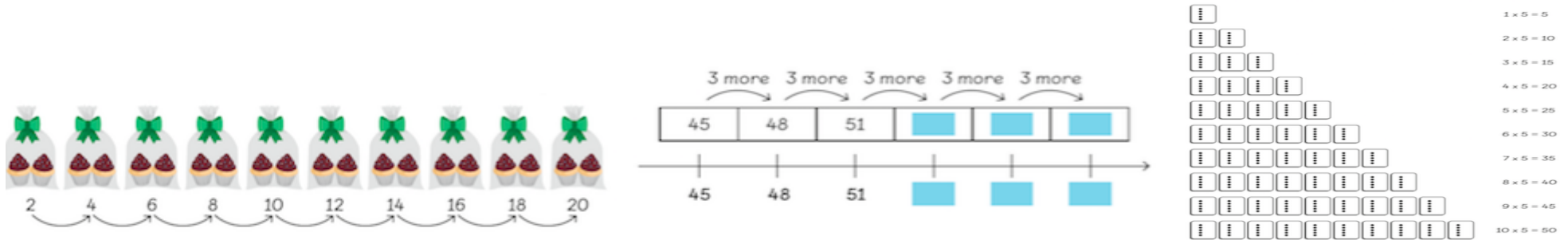
Children should use this to check **inverse** calculations. This can be used to **reason** and prove that they are correct. Using the **bars**, children need to make the link between **part-part-whole**.

Year 2 - Multiplication

Key Vocabulary

Part, whole, multiple, multiplicand, multiplier, product, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows

Skip counting in multiples of 2, 3, 5, 10 from 0



Children need to recall and use **multiplication facts** for the **multiplication tables 2, 5 and 10**. Through skip counting with different objects, children need to develop an understanding of how **groups of** items get larger with each jump and the idea of the **whole**.

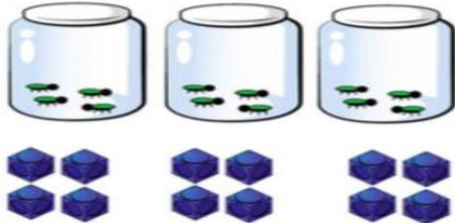
Repeated grouping / repeated addition

Repeated grouping/repeated addition

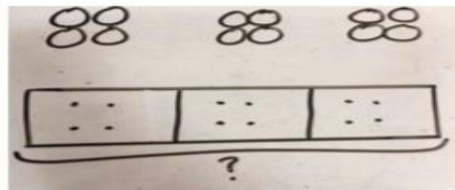
$$3 \times 4$$

$$4 + 4 + 4$$

There are 3 equal groups, with 4 in each group.



Children to represent the practical resources in a picture and use a bar model.

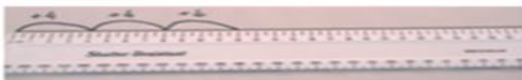


$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

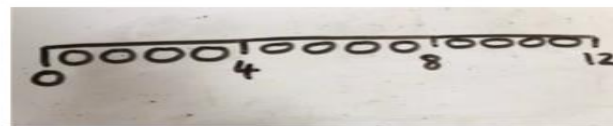
Number lines to show repeated groups-

$$3 \times 4$$



Cuisenaire rods can be used too.

Represent this pictorially alongside a number line e.g.:



Abstract number lines showing three jumps of four.

$$3 \times 4 = 12$$



With **repeated grouping**, children can build their understanding of how **multiplication increases a whole** amount.

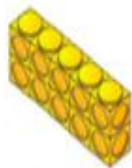
Commutativity

Use arrays to illustrate commutativity counters and other objects can also be used.

$$2 \times 5 = 5 \times 2$$

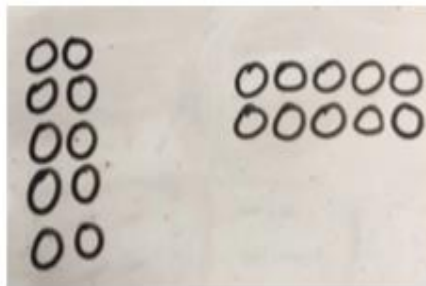


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

Children need introducing to the concept of **commutativity**. The idea what **multiplication** can be calculated either way around. The answer will be the same. **Addition** is also **commutative**. **Arrays** are the best way to explore this concept in the initial instance.

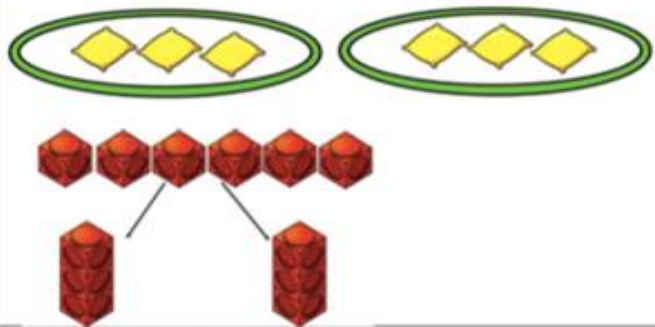
Year 2 - Division

Key Vocabulary

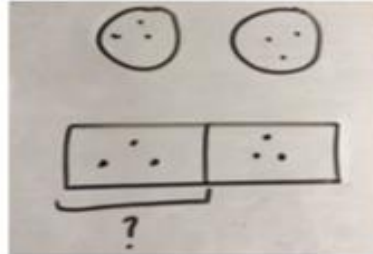
Part, whole, group in pairs, 3s ... 10s etc, equal groups of, divide, \div , divided by, divided into, remainder, dividend, divisor, quotient

Sharing into equal groups

Sharing using a range of objects.
 $6 \div 2$



Represent the sharing pictorially.



$6 \div 2 = 3$

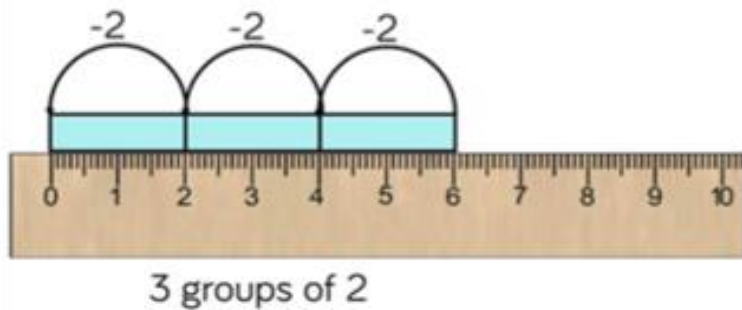
3	3
---	---

Children should also be encouraged to use their 2 times tables facts.

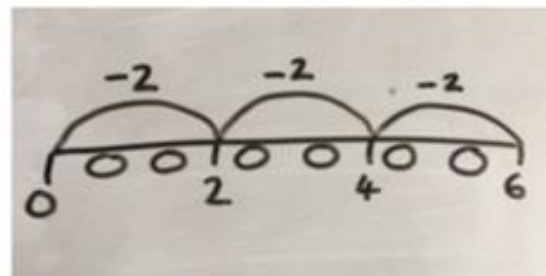
Using concrete resources, children can **group** objects to **share** them **equally**. Links can be made between **multiplication** and how the **times tables** facts relate to **groups of**. Children should also be made aware that **multiplication** and **division** are **inverse** operations.

Repeated Subtraction

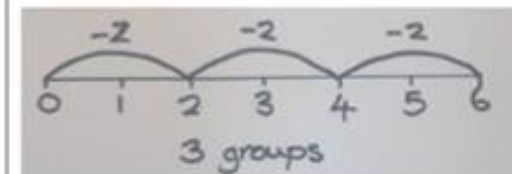
Repeated subtraction using Cuisenaire rods above a ruler.
 $6 \div 2$



Children to represent repeated subtraction pictorially.



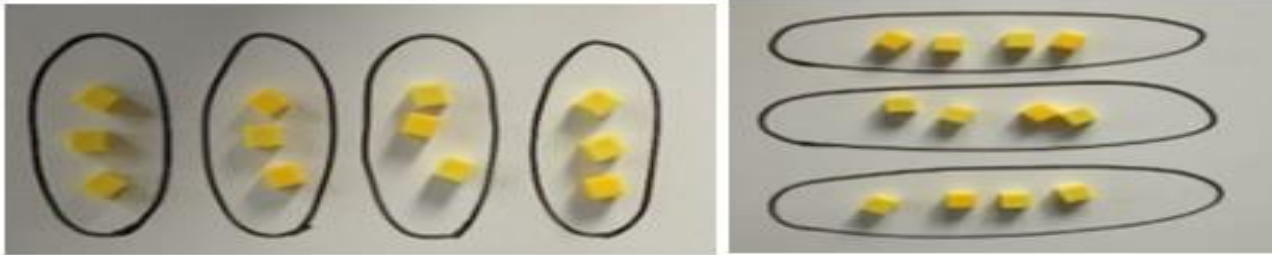
Abstract number line to represent the equal groups that have been subtracted.



Repeated subtraction allows children to use the **ordinality of number** to **share** objects **equally**. Again, this method could then be proved using **multiplication statements**.

Solve division problems in context by:

Using arrays



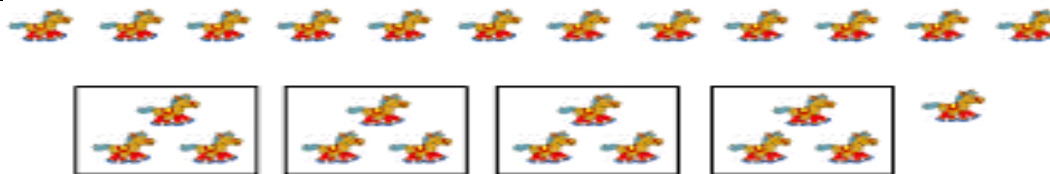
Division as grouping

Put 10 buns in groups of 2.
How many plates are there?



Once children have a firm understanding of **division** as **sharing** amounts **equally**, they should be introduced to solving problems in context. Using **arrays** ensures that children make the link between **multiplication** and **division**.

As above but working at Greater Depth with remainders



$$13 \div 4 = 3 \text{ Remainder } 1$$

To extend children's understanding and deepen their knowledge of **division**, children working at **greater depth** should be introduced to **division including remainders**. They should recognise that a **remainder** is worth something.

Year 3



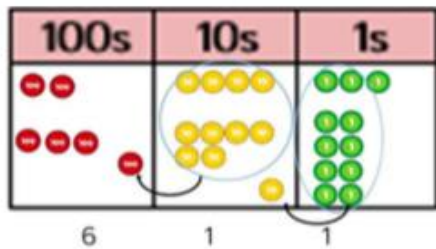
Year 3 - Addition

Key Vocabulary

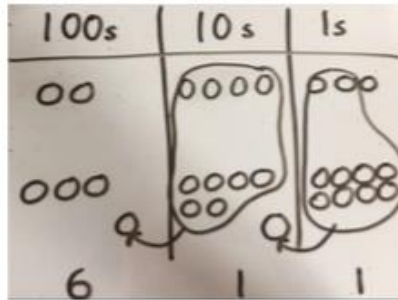
Part, whole, addend, sum, hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange - See also Y1 and Y2

Add two three-digit numbers

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

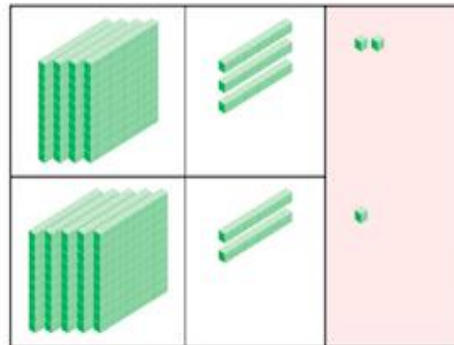


Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$$

Add the ones.
2 ones + 1 one = 3 ones



$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 4 \quad 3 \quad 2 \\ + 5 \quad 2 \quad 1 \\ \hline \quad \quad 3 \end{array}$$

Children need to use equipment first to support their understanding of **place value**. Children to progress gradually to **three digit + three digit** starting without **carrying** and gradually moving towards **carrying**.

Using the bar to find missing digits

Bar Model to support understanding of problem solving:



A man sold 230 balloons at a carnival in the morning.
He sold another 86 balloons in the evening . How
many balloons did he sell in all?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 3 - Subtraction

Key Vocabulary

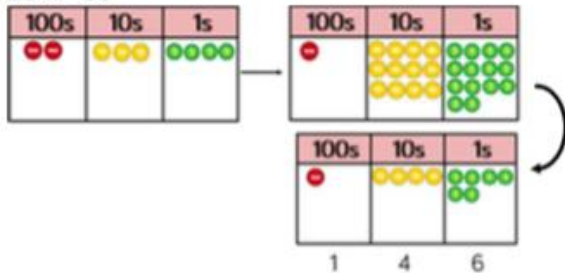
Part, whole, hundreds, tens, ones, estimate, partition, recombine, subtrahend, minuend, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

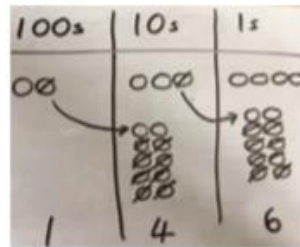
Subtract up to 3 digits from 3 digits

Column method using place value counters.

$$234 - 88$$



Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$

It is very important for children to use **dienes equipment** along with a **place value chart** to support. Only when secure with the method should **exchanging** be introduced. **Place value counters** are imperative here.

Using the bar to find missing digits

315	$315 - 185 = ?$
185 ?	$185 + ? = 315$

?	$185 + 315 = ?$
185 315	$? - 185 = 315$

It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 3 - Multiplication

Key Vocabulary

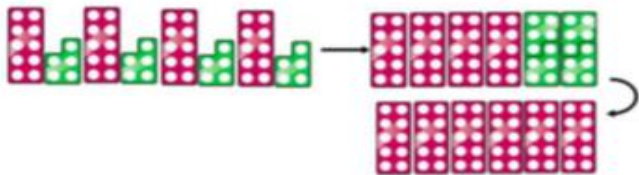
Part, whole, multiple, multiplicand, multiplier, product, factor, partition, short multiplication and inverse

Multiply a two-digit number by a one digit

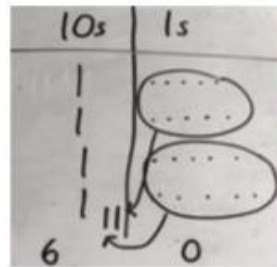
Children should be able to recall the 2, 5, 10, 3, 4 and 8 times tables.

Partition to multiply using Numicon, base 10 or Cuisenaire rods.

$$4 \times 15$$



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

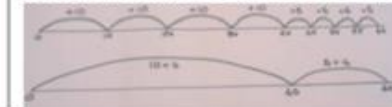
$$4 \times 15$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

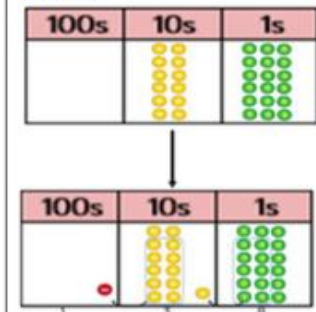
$$40 + 20 = 60$$

A number line can also be used

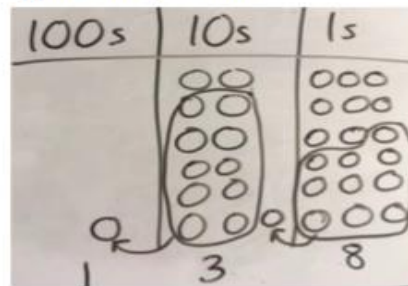


Being able to **partition** a number when **multiplying** and being able to visualise it correctly will aid **mental calculation** later on in a child's learning.

Formal column method with place value counters.



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$6 \times 23 =$$

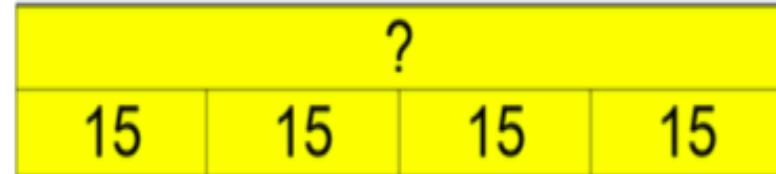
$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

As children become more confident **with multiplying** a **one digit number** by a **one digit** number, lining up the **counters** will allow children to understand that **multiplication is groups of** something and how it **increases**. This will lend itself to understanding that **division** is the **inverse operation**.

Using the bar to solve multiplication problems

4 children go to the cinema.
They each pay £15. How much
do they spend altogether?

Whole unknown



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 3 - Division

Key Vocabulary

Part, whole, divisor, dividend, quotient, inverse, remainder. Also, see Y1 and Y2

Short division

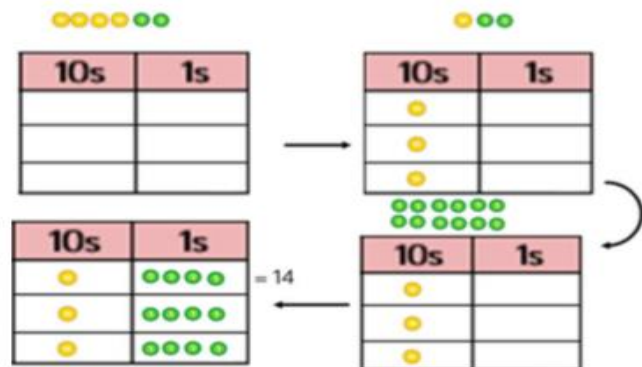
$$\begin{array}{r}
 \text{T} \qquad \text{U} \\
 \hline
 2 \qquad 3 \\
 3 \overline{) 69} \\
 \underline{6} \qquad 9 \\
 \text{•••} \qquad \text{•••••} \\
 \text{•••} \qquad \text{•••••} \\
 \text{•••} \qquad \text{•••••}
 \end{array}$$

Remind children of correct **place value**, that **69** is **equal** to **60** and **9**, but in **short division**, pose:

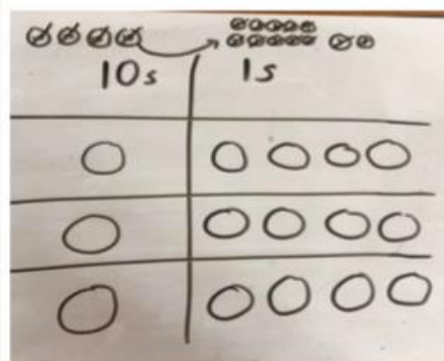
- How many 3's in 6? = 2, and record it above the **6 tens**.
- How many 3's in 9? = 3, and record it above the **9 ones**.

Sharing using place value counters (including carrying remainders within the division)

Sharing using place value counters.
 $42 \div 3 = 14$



Children to represent the place value counters pictorially.



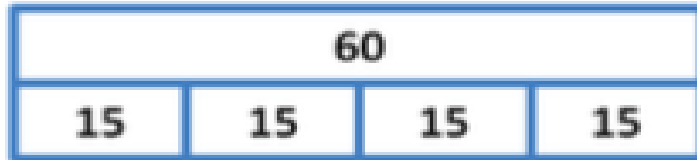
Children to be able to make sense of the place value counters and write calculations to show the process.

$$\begin{array}{l}
 42 \div 3 \\
 42 = 30 + 12 \\
 30 \div 3 = 10 \\
 12 \div 3 = 4 \\
 10 + 4 = 14
 \end{array}$$

Once children demonstrate a full understanding of **remainders**, and also the **short division** method taught, they can be taught how to use the method when **remainders** occur within the **calculation** (e.g. $42 \div 3$), and be taught to '**carry**' the **remainder** onto the next **digit**.

Using the bar to aid the solving of division problems - grouping and sharing

$$60 \div 4 = 15$$



'60 in four equal parts'

$$28 \div 7 = 4$$



'How many 7s in 28?'

It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 4

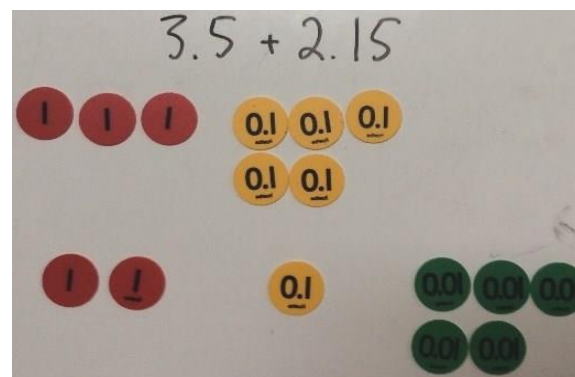
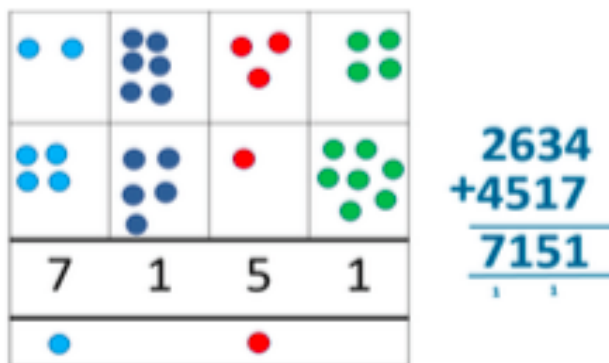


Year 4 - Addition

Key Vocabulary

Part, whole, add, addition, addend, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? Ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

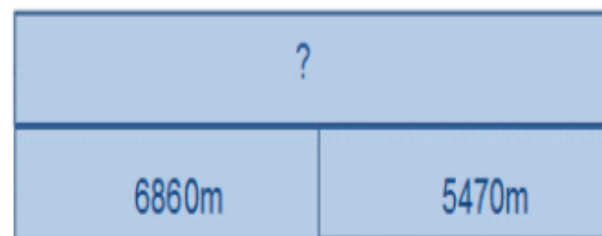
Adding numbers with up to 4 digits



Again this should start with the children using **dienes** to support them with lots of discussion about the **value of each digit**. Once children are ready they should be encouraged to **pictorially represent** the numbers they are **adding**. Children should have a firm understanding of how to **partition** the **numbers** and should be able to explain the **value of each digit**.

Using the bar to find missing digits

Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?



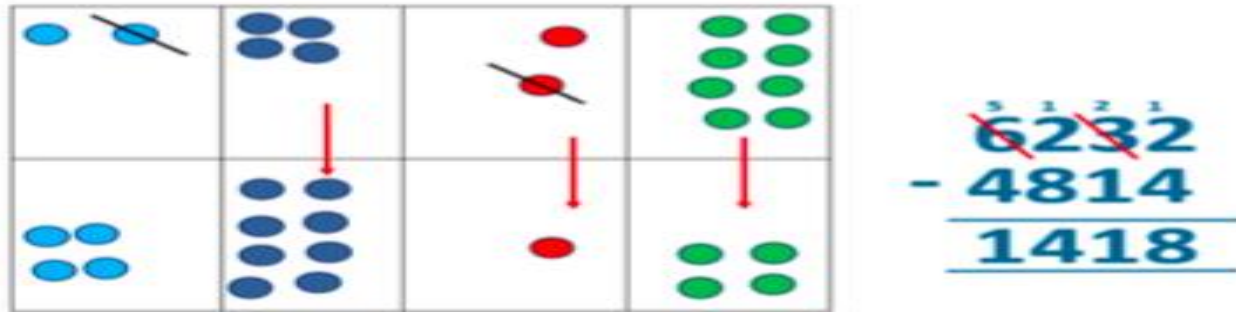
It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. This is not a form of getting the correct answer but helping to guide children to the **correct operation**.

Year 4 - Subtraction

Key Vocabulary

Part, whole, subtract, takeaway, less, minus, decrease, fewer, subtrahend, minuend, difference, how many less to make..? how much less? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many fewer? Equals sign, is the same as.

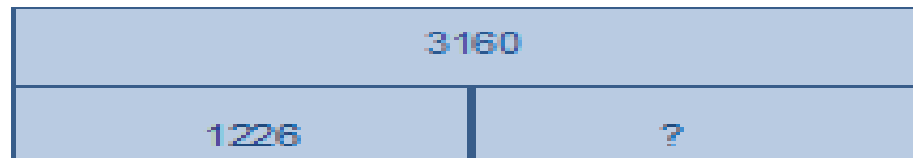
To subtract with numbers up to four digits (including exchanging when children are secure)



Children need to use **place value counters** to support their learning. All children should be able to understand what actually happens when they **exchange** and **carry** across a **subtraction**. Working out a problem one step at a time and vocalising the method should be encouraged.

Using the bar to find missing digits

There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 4 - Multiplication

Vocabulary

Part, whole, factor, product, multiplier, multiplicand

Multiplying both two and three digits by a one digit number using place value counters

Children to know all times tables to 12×12



$$\begin{array}{r} 473 \\ \times 2 \\ \hline \end{array}$$

Handwritten multiplication showing $235 \times 6 = 1410$. The calculation is written as:

$$\begin{array}{r} 235 \\ \times 6 \\ \hline 1410 \end{array}$$

Expanded multiplication using place value counters is instrumental in children understanding the place value of each digit as they multiply them together. It will aid overall understanding that by multiplying a number makes it bigger.

Multiplying using the bar

A computer costs 5 times as much as a television. The television costs £429.

Cost of the computer



How much does the computer cost?

It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

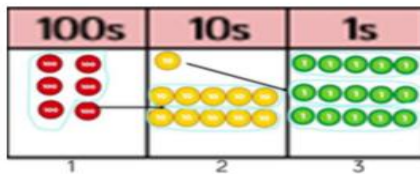
Year 4 - Division

Key Vocabulary

Part, whole, see years 1-3, divide, divided by, divisible by, divided into, share between, groups of, factor, factor pair, multiple, times as (big, long, wide ...etc), equals, remainder, quotient, dividend, divisor and inverse

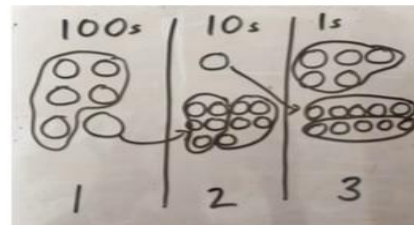
Dividing up to three digit numbers by a one digit number using short division

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

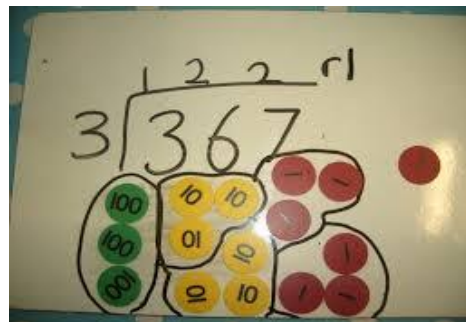


Children to the calculation using the short division scaffold.

$$5 \overline{) 615} \begin{matrix} 123 \\ \\ \end{matrix}$$

Only when the children are secure with **dividing a two digit number** should they move onto a **3 digit number**. When children are working through the **division using counters**, they should be shown the **written method** side-by-side. This will aid their understanding of how the two are connected and what it actually happening when they write the numbers above a **short division written method**.

Dividing up to three digit numbers by a one digit number using short division with remainders



	H	T	U	
	0	2	5	r1
5	1	2	6	

When children are secure, they should be encouraged to recognise that a **remainder** is relevant and needs to be counted as part of the answer.

Dividing using the bar

Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 5



Year 5 - Addition

Key Vocabulary

Part, whole, addend, sum, tens of thousands boundary - Also see previous years

Adding numbers with more than 4 digits including decimals

$$\begin{array}{r} 45867 \\ + 32192 \\ \hline 78059 \end{array}$$

$$\begin{array}{r} 3.17 \\ + 4.25 \\ \hline 7.42 \end{array}$$

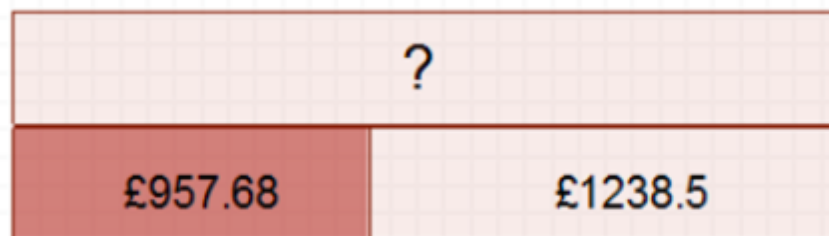
$$\begin{array}{r} 3.460 \\ + 3.792 \\ \hline 10.252 \end{array}$$

Zero used as a place value holder.

Using **place value charts** are key to this as well as **place value counters** to help with the **decimals**. Children should be able to explain the **value of each digit** and make links between **greater than** and **less than**. Children should also be encouraged to relate the written method back to **part-part-whole**.

Using the bar to find missing digits

MacDonalds sold £9957.68 worth of hamburgers and £1238.5 worth of chicken nuggets. How much money did they take altogether?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. This is not a form of getting the correct answer but helping to guide children to the correct operation.

Year 5 - Subtraction

Key Vocabulary

Part, whole, tens of thousands boundary, subtrahend, minuend, difference

Also see previous years

Subtract with at least four digit numbers including two decimal places

	2	10	10	5	6
-		2	1	2	8
	2	8	9	2	8

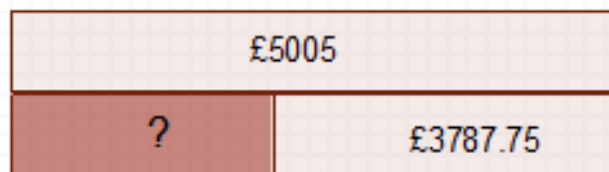
Handwritten calculation showing the subtraction of 2.91 from 4.63. The result is 1.72. The calculation is written as follows:

$$\begin{array}{r} 4.63 - 2.91 = \\ \underline{4} \cdot 63 \\ - 2.91 \\ \hline 1.72 \end{array}$$

Include **money**, **measures** and **decimals** ensuring that children do this **practically** before the **abstract**. Understanding the **value of money** and how it related to **the place value** of a calculation is important. **Subtract with decimal values**, including **mixtures of integers and decimals, aligning the decimal point**.

Using the bar to find missing digits

A whole to Lapland costs £5005 for a family of four, the Smith's have only saved £3787.75, how much money do they still need to find?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 5 - Multiplication

Key Vocabulary

Part, whole, cube numbers, prime numbers, square numbers, common factors, prime number, prime factors and composite numbers, factor, multiplier, multiplicand, product

Multiplying up to four digit numbers by two digits using long multiplication

	3	6	5	2
x				8
<hr/>				
2	9	2	1	6
	5	4	1	

	1	2	3	4
x			1	6
<hr/>				
7	4	0	4	
1	2	3	4	0
<hr/>				
1	9	7	4	4

(1234 × 6)

(1234 × 10)

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .
To get 2480 they have solved 20×124 .

	1	2	4
x		2	6
<hr/>			
7	4	4	
2	4	8	0
<hr/>			
3	2	2	4
<hr/>			
1	1		

Answer: 3224

Children need to be taught to **approximate** first, e.g. for 72×38 , they will use rounding: 72×38 is approximately $70 \times 40 = 2800$, and use the **approximation** to check the reasonableness of their answer.

Using the bar to support multiplication

The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.

Year 5 - Division

Key Vocabulary

Part, whole, common factors, prime number, prime factors, composite numbers, short division, square number, cube number, inverse, power of, divisor, dividend, quotient, remainder. Also see year 4

Diving with up to four digit numbers by one digit including numbers where remainders are left

A handwritten short division problem on a grid background. The divisor is 8, the dividend is 6635, and the quotient is 829 with a remainder of 5. The calculation is shown as follows:

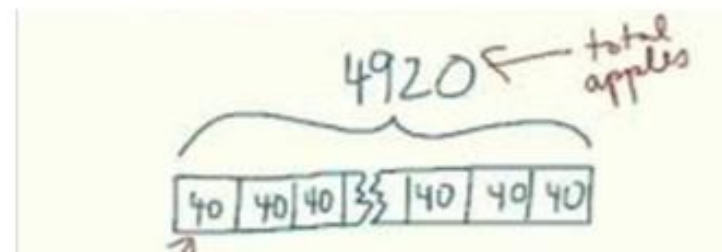
$$\begin{array}{r} 8 \overline{) 6635} \\ \underline{8} \\ 66 \\ \underline{56} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 4 \\ \underline{4} \\ 0 \\ \underline{0} \\ 5 \end{array}$$

Short division with remainders: Now that pupils are introduced to examples that give rise to **remainder** answers, **division** needs to have a real life problem solving context, where **pupils consider the meaning of the remainder** and **how** to express it, ie. as a **fraction**, a **decimal**, or as a **rounded number** or **value**, depending upon the context of the problem.

Using the bar to support division problems

Bar Model to support understanding of problem solving:

Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need?



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Year 6



Year 6 - Multiplication

Key Vocabulary

Part, whole, common factors, multiplicand, multiplier, product. See previous years.

Short and long multiplication with up to two decimal places

$$\begin{array}{r}
 2368 \\
 \times 34 \\
 \hline
 9472 \\
 \overset{\pm}{1}\overset{\pm}{2}\overset{\pm}{3} \\
 71040 \\
 \hline
 \overset{\pm}{8}\overset{\pm}{0}\overset{\pm}{5}\overset{\pm}{1}\overset{\pm}{2}
 \end{array}$$

A handwritten multiplication problem on a grid background. The multiplicand is 3.19 and the multiplier is 8. The product is 25.52. The decimal point in the product is aligned with the decimal point in the multiplicand. There are small arrows pointing to the decimal places in the product.

Children need to be exposed to a wide range of **multiplication problems** including up to **2 decimal places**.

Using the bar for multiplication

If 5 friends went on holiday and each paid £579.75 what was the total cost of the holiday?

Cost of the holiday



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Long Division using skeleton tables

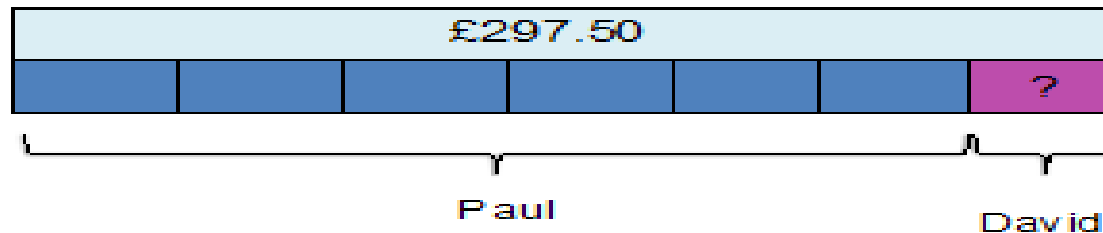
				6	3
2	3	1	4	4	9
	-	1	3	8	↓
			6	9	
			-	6	9
					0

1	-	23
2	-	46
3	-	69
4	-	92
5	-	115
6	-	138
7	-	161
8	-	184
9	-	207
10	-	230

Children should be introduced to **long division** and **dividing by 2 digits** through using **skeleton tables**. This method will allow children to **estimate** an answer using their **times tables** before they solve the method properly.

Using the bar for division

Paul and David hire a car together at a cost of £297.50. Paul pays 6 times more than David. How much does David pay?



It is important for children to use **the bar** in this way to encourage the use of it to aid with **problem solving**. The **bar** is a great way for children **to pictorially** record and visualise what a problem is asking.